2-1 Using Inductive Reasoning to Make Conjectures

Warm Up
Complete each sentence.

1. ___ points are points that lie on the same line.
   Collinear

2. ___ points are points that lie in the same plane.
   Coplanar

3. The sum of the measures of two ___ angles is 90°.
   complementary
2-1 Using Inductive Reasoning to Make Conjectures

**Objectives**

Use inductive reasoning to identify patterns and make conjectures.

Find counterexamples to disprove conjectures.
Vocabulary

inductive reasoning
conjecture
counterexample
Example 1A: Identifying a Pattern

Find the next item in the pattern.

January, March, May, ...

*Alternating months of the year make up the pattern.*

The next month is July.
Example 1B: Identifying a Pattern

Find the next item in the pattern.

7, 14, 21, 28, ...

*Multiples of 7 make up the pattern.*

The next multiple is 35.
Example 1C: Identifying a Pattern

Find the next item in the pattern.

In this pattern, the figure rotates 90° counter-clockwise each time.

The next figure is △.
Check It Out! Example 1

Find the next item in the pattern 0.4, 0.04, 0.004, ...

When reading the pattern from left to right, the next item in the pattern has one more zero after the decimal point.

The next item would have 3 zeros after the decimal point, or 0.0004.
When several examples form a pattern and you assume the pattern will continue, you are applying **inductive reasoning**. **Inductive reasoning** is the process of reasoning that a rule or statement is true because specific cases are true. You may use inductive reasoning to draw a conclusion from a pattern. A statement you believe to be true based on inductive reasoning is called a **conjecture**.
Example 2A: Making a Conjecture

Complete the conjecture.

The sum of two positive numbers is ___.

*List some examples and look for a pattern.*

1 + 1 = 2  
3.14 + 0.01 = 3.15  
3,900 + 1,000,017 = 1,003,917

The sum of two positive numbers is **positive**.
Example 2B: Making a Conjecture

Complete the conjecture.

The number of lines formed by 4 points, no three of which are collinear, is $\text{?}$. 

*Draw four points. Make sure no three points are collinear.*

*Count the number of lines formed:*

\[
AB \quad AC \quad AD \quad BC \quad BD \quad CD
\]

The number of lines formed by four points, no three of which are collinear, is 6.
Complete the conjecture.

The product of two odd numbers is ____.

_List some examples and look for a pattern._

1 \times 1 = 1 \quad 3 \times 3 = 9 \quad 5 \times 7 = 35

The product of two odd numbers is odd.
To show that a conjecture is always true, you must prove it.

To show that a conjecture is false, you have to find only one example in which the conjecture is not true. This case is called a \textbf{counterexample}.

A counterexample can be a drawing, a statement, or a number.
# Using Inductive Reasoning to Make Conjectures

## Inductive Reasoning

1. Look for a pattern.
2. Make a conjecture.
3. Prove the conjecture or find a counterexample.
Example 4A: Finding a Counterexample

Show that the conjecture is false by finding a counterexample.
For every integer \( n \), \( n^3 \) is positive.

Pick integers and substitute them into the expression to see if the conjecture holds.

Let \( n = 1 \). Since \( n^3 = 1 \) and \( 1 > 0 \), the conjecture holds.

Let \( n = -3 \). Since \( n^3 = -27 \) and \( -27 \leq 0 \), the conjecture is false.

\( n = -3 \) is a counterexample.
Example 4B: Finding a Counterexample

Show that the conjecture is false by finding a counterexample.

Two complementary angles are not congruent.

\[45° + 45° = 90°\]

*If the two congruent angles both measure 45°, the conjecture is false.*
Example 4C: Finding a Counterexample

Show that the conjecture is false by finding a counterexample.

The monthly high temperature in Abilene is never below 90°F for two months in a row.

<table>
<thead>
<tr>
<th>Monthly High Temperatures (°F) in Abilene, Texas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
</tr>
<tr>
<td>88</td>
</tr>
</tbody>
</table>

The monthly high temperatures in January and February were 88°F and 89°F, so the conjecture is false.
Show that the conjecture is false by finding a counterexample.
For any real number $x$, $x^2 \geq x$.

Let $x = \frac{1}{2}$.

Since $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$, $\frac{1}{4} \geq \frac{1}{2}$

The conjecture is false.
Show that the conjecture is false by finding a counterexample.

Supplementary angles are adjacent.

The supplementary angles are not adjacent, so the conjecture is false.
Lesson Quiz

Find the next item in each pattern.

1. 0.7, 0.07, 0.007, ...  
   0.0007

2. 0.0007

Determine if each conjecture is true. If false, give a counterexample.

3. The quotient of two negative numbers is a positive number.  
   true

4. Every prime number is odd.  
   false; 2

5. Two supplementary angles are not congruent.  
   false; 90° and 90°

6. The square of an odd integer is odd.  
   true
Warm Up
Determine if each statement is true or false.

1. The measure of an obtuse angle is less than 90°.  
   \[ \boxed{F} \]

2. All perfect-square numbers are positive.  
   \[ \boxed{F} \]

3. Every prime number is odd.  
   \[ \boxed{T} \]

4. Any three points are coplanar.  
   \[ \boxed{F} \]
Objectives

Identify, write, and analyze the truth value of conditional statements.

Write the inverse, converse, and contrapositive of a conditional statement.
2-2 Conditional Statements

Vocabulary

conditional statement
hypothesis
conclusion
truth value
negation
converse
inverse
contrapositive
logically equivalent statements
By phrasing a conjecture as an if-then statement, you can quickly identify its hypothesis and conclusion.
Example 1: Identifying the Parts of a Conditional Statement

Identify the hypothesis and conclusion of each conditional.

A. If today is Thanksgiving Day, then today is Thursday.
   Hypothesis: Today is Thanksgiving Day. 
   Conclusion: Today is Thursday.

B. A number is a rational number if it is an integer.
   Hypothesis: A number is an integer. 
   Conclusion: The number is a rational number.
Identify the hypothesis and conclusion of the statement.

"A number is divisible by 3 if it is divisible by 6."

Hypothesis: A number is divisible by 6.
Conclusion: A number is divisible by 3.
“If $p$, then $q$" can also be written as “if $p$, $q$,” “$q$, if $p$,” “$p$ implies $q$,” and “$p$ only if $q$."

Many sentences without the words *if* and *then* can be written as conditionals. To do so, identify the sentence’s hypothesis and conclusion by figuring out which part of the statement depends on the other.
2-2 Conditional Statements

Example 2A: Writing a Conditional Statement

Write a conditional statement from the following.

An obtuse triangle has exactly one obtuse angle.

An obtuse triangle has exactly one obtuse angle. Identify the hypothesis and the conclusion.

If a triangle is obtuse, then it has exactly one obtuse angle.
Example 2B: Writing a Conditional Statement

Write a conditional statement from the following.

If an animal is a blue jay, then it is a bird.

The inner oval represents the hypothesis, and the outer oval represents the conclusion.
Check It Out! Example 2

Write a conditional statement from the sentence “Two angles that are complementary are acute.”

Two angles that are complementary are acute.

If two angles are complementary, then they are acute.
A conditional statement has a **truth value** of either true (T) or false (F). It is false only when the hypothesis is true and the conclusion is false.

To show that a conditional statement is false, you need to find only one counterexample where the hypothesis is true and the conclusion is false.
Determine if the conditional is true. If false, give a counterexample.

If this month is August, then next month is September.

When the hypothesis is true, the conclusion is also true because September follows August. So the conditional is true.
Example 3B: Analyzing the Truth Value of a Conditional Statement

Determine if the conditional is true. If false, give a counterexample.

If two angles are acute, then they are congruent.

You can have acute angles with measures of $80^\circ$ and $30^\circ$. In this case, the hypothesis is true, but the conclusion is false.

Since you can find a counterexample, the conditional is false.
Example 3C: Analyzing the Truth Value of a Conditional Statement

Determine if the conditional is true. If false, give a counterexample.

If an even number greater than 2 is prime, then $5 + 4 = 8$.

An even number greater than 2 will never be prime, so the hypothesis is false. $5 + 4$ is not equal to 8, so the conclusion is false. However, the conditional is true because the hypothesis is false.
Check It Out! Example 3

Determine if the conditional “If a number is odd, then it is divisible by 3” is true. If false, give a counterexample.

An example of an odd number is 7. It is not divisible by 3. In this case, the hypothesis is true, but the conclusion is false. Since you can find a counterexample, the conditional is false.
Conditional Statements

Remember!

If the hypothesis is false, the conditional statement is true, regardless of the truth value of the conclusion.
The **negation** of statement $p$ is “not $p$,” written as $\sim p$. The negation of a true statement is false, and the negation of a false statement is true.
### Related Conditionals

<table>
<thead>
<tr>
<th>Definition</th>
<th>Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>A conditional is a statement that can be written in the form &quot;If $p$, then $q$.&quot;</td>
<td>$p \rightarrow q$</td>
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### Conditional Statements

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<td>The <strong>inverse</strong> is the statement formed by negating the hypothesis and conclusion.</td>
<td>$\sim p \rightarrow \sim q$</td>
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<table>
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<tr>
<th>Definition</th>
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</tr>
</thead>
<tbody>
<tr>
<td>The <strong>contrapositive</strong> is the statement formed by both exchanging and negating the hypothesis and conclusion.</td>
<td>( \sim q \rightarrow \sim p )</td>
</tr>
</tbody>
</table>
Example 4: Biology Application

Write the converse, inverse, and contrapositive of the conditional statement. Use the Science Fact to find the truth value of each.

*If an animal is an adult insect, then it has six legs.*

**Science Fact**

Adult insects have six legs.
No other animals have six legs.
Example 4: Biology Application

*If an animal is an adult insect, then it has six legs.*

Converse: If an animal has six legs, then it is an adult insect.

No other animals have six legs so the converse is true.

Inverse: If an animal is not an adult insect, then it does not have six legs.

No other animals have six legs so the converse is true.

Contrapositive: If an animal does not have six legs, then it is not an adult insect.

Adult insects must have six legs. So the contrapositive is true.
Check It Out! Example 4
Write the converse, inverse, and contrapositive of the conditional statement “If an animal is a cat, then it has four paws.” Find the truth value of each.

If an animal is a cat, then it has four paws.
Check It Out! Example 4

*If an animal is a cat, then it has four paws.*

Converse: If an animal has 4 paws, then it is a cat.

There are other animals that have 4 paws that are not cats, so the converse is false.

Inverse: If an animal is not a cat, then it does not have 4 paws.

There are animals that are not cats that have 4 paws, so the inverse is false.

Contrapositive: If an animal does not have 4 paws, then it is not a cat; True.

Cats have 4 paws, so the contrapositive is true.
Related conditional statements that have the same truth value are called **logically equivalent statements**. A conditional and its contrapositive are logically equivalent, and so are the converse and inverse.
The logical equivalence of a conditional and its contrapositive is known as the Law of Contrapositive.
Lesson Quiz: Part I

Identify the hypothesis and conclusion of each conditional.

1. A triangle with one right angle is a right triangle.
   H: A triangle has one right angle.
   C: The triangle is a right triangle.

2. All even numbers are divisible by 2.
   H: A number is even.
   C: The number is divisible by 2.

3. Determine if the statement “If $n^2 = 144$, then $n = 12$” is true. If false, give a counterexample.
   False; $n = -12$. 
Lesson Quiz: Part II

Identify the hypothesis and conclusion of each conditional.

4. Write the converse, inverse, and contrapositive of the conditional statement “If Maria’s birthday is February 29, then she was born in a leap year.” Find the truth value of each.

Converse: If Maria was born in a leap year, then her birthday is February 29; False.
Inverse: If Maria’s birthday is not February 29, then she was not born in a leap year; False.
Contrapositive: If Maria was not born in a leap year, then her birthday is not February 29; True.
Warm Up
Identify the hypothesis and conclusion of each conditional.

1. A mapping that is a reflection is a type of transformation.  
   H: A mapping is a reflection.  
   C: The mapping is a transformation.

2. The quotient of two negative numbers is positive.  
   H: Two numbers are negative.  
   C: The quotient is positive.

3. Determine if the conditional “If \( x \) is a number then \( |x| > 0 \)” is true. If false, give a counterexample.  
   F; \( x = 0 \).
Objective

Apply the Law of Detachment and the Law of Syllogism in logical reasoning.
Vocabulary

deductive reasoning
**Deductive reasoning** is the process of using logic to draw conclusions from given facts, definitions, and properties.
Is the conclusion a result of inductive or deductive reasoning?

There is a myth that you can balance an egg on its end only on the spring equinox. A person was able to balance an egg on July 8, September 21, and December 19. Therefore this myth is false.

Since the conclusion is based on a pattern of observations, it is a result of inductive reasoning.
Example 1B: Media Application

Is the conclusion a result of inductive or deductive reasoning?

There is a myth that the Great Wall of China is the only man-made object visible from the Moon. The Great Wall is barely visible in photographs taken from 180 miles above Earth. The Moon is about 237,000 miles from Earth. Therefore, the myth cannot be true.

The conclusion is based on logical reasoning from scientific research. It is a result of deductive reasoning.
There is a myth that an eelskin wallet will demagnetize credit cards because the skin of the electric eels used to make the wallet holds an electric charge. However, eelskin products are not made from electric eels. Therefore, the myth cannot be true. Is this conclusion a result of inductive or deductive reasoning?

The conclusion is based on logical reasoning from scientific research. It is a result of deductive reasoning.
In deductive reasoning, if the given facts are true and you apply the correct logic, then the conclusion must be true. The Law of Detachment is one valid form of deductive reasoning.

**Law of Detachment**

If $p \rightarrow q$ is a true statement and $p$ is true, then $q$ is true.
Example 2A: Verifying Conjectures by Using the Law of Detachment

Determine if the conjecture is valid by the Law of Detachment.

Given: If the side lengths of a triangle are 5 cm, 12 cm, and 13 cm, then the area of the triangle is $30 \text{ cm}^2$. The area of $\triangle PQR$ is $30 \text{ cm}^2$.

Conjecture: The side lengths of $\triangle PQR$ are 5cm, 12 cm, and 13 cm.
Example 2A: Verifying Conjectures by Using the Law of Detachment Continued

Identify the **hypothesis** and **conclusion** in the given conditional.

**If the side lengths of a triangle are 5 cm, 12 cm, and 13 cm, then the area of the triangle is 30 cm\(^2\).**

The given statement “The area of \(\triangle PQR\) is 30 cm\(^2\)” matches the conclusion of a true conditional. But this does not mean the hypothesis is true. The dimensions of the triangle could be different. So the conjecture is not valid.
Example 2B: Verifying Conjectures by Using the Law of Detachment

Determine if the conjecture is valid by the Law of Detachment.

Given: In the World Series, if a team wins four games, then the team wins the series. The Red Sox won four games in the 2004 World Series.

Conjecture: The Red Sox won the 2004 World Series.
Example 2B: Verifying Conjectures by Using the Law of Detachment Continued

Identify the **hypothesis** and **conclusion** in the given conditional.

**In the World Series, if a team wins four games, then the team wins the series.**

The statement “The Red Sox won four games in the 2004 World Series” matches the hypothesis of a true conditional. By the Law of Detachment, the Red Sox won the 2004 World Series. The conjecture is valid.
Determine if the conjecture is valid by the Law of Detachment.

Given: If a student passes his classes, the student is eligible to play sports. Ramon passed his classes.

Conjecture: Ramon is eligible to play sports.
Identify the **hypothesis** and **conclusion** in the given conditional.

**If a student passes his classes, then the student is eligible to play sports.**

The statement “Ramon passed his classes” matches the hypothesis of a true conditional. By the Law of Detachment, Ramon is eligible to play sports. The conjecture is valid.
Another valid form of deductive reasoning is the Law of Syllogism. It allows you to draw conclusions from two conditional statements when the conclusion of one is the hypothesis of the other.

**Law of Syllogism**

If $p \rightarrow q$ and $q \rightarrow r$ are true statements, then $p \rightarrow r$ is a true statement.
Example 3A: Verifying Conjectures by Using the Law of Syllogism

Determine if the conjecture is valid by the Law of Syllogism.

Given: If a figure is a kite, then it is a quadrilateral. If a figure is a quadrilateral, then it is a polygon.

Conjecture: If a figure is a kite, then it is a polygon.
Example 3A: Verifying Conjectures by Using the Law of Syllogism Continued

Let $p$, $q$, and $r$ represent the following.

$p$: A figure is a kite.
$q$: A figure is a quadrilateral.
$r$: A figure is a polygon.

You are given that $p \rightarrow q$ and $q \rightarrow r$.

Since $q$ is the conclusion of the first conditional and the hypothesis of the second conditional, you can conclude that $p \rightarrow r$. The conjecture is valid by Law of Syllogism.
Example 3B: Verifying Conjectures by Using the Law of Syllogism

Determine if the conjecture is valid by the Law of Syllogism.

Given: If a number is divisible by 2, then it is even. If a number is even, then it is an integer.

Conjecture: If a number is an integer, then it is divisible by 2.
Example 3B: Verifying Conjectures by Using the Law of Syllogism Continued

Let \( x \), \( y \), and \( z \) represent the following.

\( x \): A number is divisible by 2.
\( y \): A number is even.
\( z \): A number is an integer.

You are given that \( x \rightarrow y \) and \( y \rightarrow z \). The Law of Syllogism cannot be used to deduce that \( z \rightarrow x \). The conclusion is not valid.
Determine if the conjecture is valid by the Law of Syllogism.

Given: If an animal is a mammal, then it has hair. If an animal is a dog, then it is a mammal.

Conjecture: If an animal is a dog, then it has hair.
Let \( x, y, \) and \( z \) represent the following.

\( x: \) An animal is a mammal.
\( y: \) An animal has hair.
\( z: \) An animal is a dog.

You are given that \( x \rightarrow y \) and \( z \rightarrow x \).

Since \( x \) is the conclusion of the second conditional and the hypothesis of the first conditional, you can conclude that \( z \rightarrow y \). The conjecture is valid by Law of Syllogism.
Example 4: Applying the Laws of Deductive Reasoning

Draw a conclusion from the given information.

A. Given: If $2y = 4$, then $z = -1$. If $x + 3 = 12$, then $2y = 4$. $x + 3 = 12$

Conclusion: $z = -1$.

B. If the sum of the measures of two angles is $180^\circ$, then the angles are supplementary. If two angles are supplementary, they are not angles of a triangle. $m\angle A = 135^\circ$, and $m\angle B = 45^\circ$.

Conclusion: $\angle A$ and $\angle B$ are not angles of a triangle.
Draw a conclusion from the given information.

Given: If a polygon is a triangle, then it has three sides.

If a polygon has three sides, then it is not a quadrilateral. Polygon $P$ is a triangle.

Conclusion: Polygon $P$ is not a quadrilateral.
Lesson Quiz: Part I

Is the conclusion a result of inductive or deductive reasoning?

1. At Reagan High School, students must pass Geometry before they take Algebra 2. Emily is in Algebra 2, so she must have passed Geometry.

   deductive reasoning
Determine if each conjecture is valid?

2. Given: If $n$ is a natural number, then $n$ is an integer. If $n$ is an integer, then $n$ is a rational number. 0.875 is a rational number.

Conjecture: 0.875 is a natural number. not valid

3. Given: If an American citizen is at least 18 years old, then he or she is eligible to vote. Anna is a 20-year-old American citizen.

Conjecture: Anna is eligible to vote. valid
Warm Up
Write a conditional statement from each of the following.

1. The intersection of two lines is a point.
   If two lines intersect, then they intersect in a point.

2. An odd number is one more than a multiple of 2.
   If a number is odd, then it is one more than a multiple of 2.

3. Write the converse of the conditional “If Pedro lives in Chicago, then he lives in Illinois.” Find its truth value.
   If Pedro lives in Illinois, then he lives in Chicago; False.
Objective

Write and analyze biconditional statements.
2-4 Biconditional Statements and Definitions

Vocabulary

biconditional statement
definition
polygon
triangle
quadrilateral
When you combine a conditional statement and its converse, you create a *biconditional statement*.

A **biconditional statement** is a statement that can be written in the form “*p* if and only if *q.*” This means “if *p*, then *q*” and “if *q*, then *p.*”
The biconditional “$p$ if and only if $q$” can also be written as “$p$ iff $q$” or $p \iff q$. 

Writing Math
Example 1A: Identifying the Conditionals within a Biconditional Statement

Write the conditional statement and converse within the biconditional.

An angle is obtuse if and only if its measure is greater than 90° and less than 180°.

Let $p$ and $q$ represent the following.

$p$: An angle is obtuse.
$q$: An angle’s measure is greater than 90° and less than 180°.
Example 1A Continued

Let \( p \) and \( q \) represent the following.

\( p: \) An angle is obtuse.
\( q: \) An angle’s measure is greater than 90° and less than 180°.

The two parts of the biconditional \( p \leftrightarrow q \) are \( p \rightarrow q \) and \( q \leftarrow p \).

Conditional: If an \( \angle \) is obtuse, then its measure is greater than 90° and less than 180°.

Converse: If an angle's measure is greater than 90° and less than 180°, then it is obtuse.
2-4  Biconditional Statements and Definitions

Example 1B: Identifying the Conditionals within a Biconditional Statement

Write the conditional statement and converse within the biconditional.

A solution is neutral ↔ its pH is 7.

Let $x$ and $y$ represent the following.

$x$: A solution is neutral.

$y$: A solution’s pH is 7.
Let \( x \) and \( y \) represent the following.

\( x \): A solution is neutral.
\( y \): A solution’s pH is 7.

The two parts of the biconditional \( x \leftrightarrow y \) are \( x \rightarrow y \) and \( y \rightarrow x \).

Conditional: **If a solution is neutral, then its pH is 7.**

Converse: **If a solution’s pH is 7, then it is neutral.**
Check It Out! Example 1a

Write the conditional statement and converse within the biconditional.

An angle is acute iff its measure is greater than $0^\circ$ and less than $90^\circ$.
Let $x$ and $y$ represent the following.

$x$: An angle is acute.

$y$: An angle has a measure that is greater than $0^\circ$ and less than $90^\circ$. 
Check It Out! Example 1a Continued

Let $x$ and $y$ represent the following.

$x$: An angle is acute.

$y$: An angle has a measure that is greater than 0° and less than 90°.

The two parts of the biconditional $x \leftrightarrow y$ are $x \rightarrow y$ and $y \rightarrow x$.

Conditional: If an angle is acute, then its measure is greater than 0° and less than 90°.

Converse: If an angle’s measure is greater than 0° and less than 90°, then the angle is acute.
Check It Out! Example 1b

Write the conditional statement and converse within the biconditional.

Cho is a member if and only if he has paid the $5 dues.

Let $x$ and $y$ represent the following.

$x$: Cho is a member.

$y$: Cho has paid his $5 dues.

The two parts of the biconditional $x \leftrightarrow y$ are $x \rightarrow y$ and $y \rightarrow x$.

Conditional: If Cho is a member, then he has paid the $5 dues.

Converse: If Cho has paid the $5 dues, then he is a member.
Example 2: Identifying the Conditionals within a Biconditional Statement

For each conditional, write the converse and a biconditional statement.

A. If $5x - 8 = 37$, then $x = 9$.
   Converse: If $x = 9$, then $5x - 8 = 37$.
   Biconditional: $5x - 8 = 37$ if and only if $x = 9$.

B. If two angles have the same measure, then they are congruent.
   Converse: If two angles are congruent, then they have the same measure.
   Biconditional: Two angles have the same measure if and only if they are congruent.
Check It Out! Example 2a

For the conditional, write the converse and a biconditional statement.

If the date is July 4th, then it is Independence Day.

Converse: If it is Independence Day, then the date is July 4th.

Biconditional: It is July 4th if and only if it is Independence Day.
Check It Out! Example 2b

For the conditional, write the converse and a biconditional statement.

If points lie on the same line, then they are collinear.

Converse: If points are collinear, then they lie on the same line.

Biconditional: Points lie on the same line if and only if they are collinear.
For a biconditional statement to be true, both the conditional statement and its converse must be true. If either the conditional or the converse is false, then the biconditional statement is false.
Example 3A: Analyzing the Truth Value of a Biconditional Statement

Determine if the biconditional is true. If false, give a counterexample.

A rectangle has side lengths of 12 cm and 25 cm if and only if its area is 300 cm².
Example 3A: Analyzing the Truth Value of a Biconditional Statement

Conditional: If a rectangle has side lengths of 12 cm and 25 cm, then its area is 300 cm².

The conditional is true.

Converse: If a rectangle’s area is 300 cm², then it has side lengths of 12 cm and 25 cm.

The converse is false.

If a rectangle’s area is 300 cm², it could have side lengths of 10 cm and 30 cm. Because the converse is false, the biconditional is false.
Example 3B: Analyzing the Truth Value of a Biconditional Statement

Determine if the biconditional is true. If false, give a counterexample.

A natural number $n$ is odd $\iff n^2$ is odd.

Conditional: If a natural number $n$ is odd, then $n^2$ is odd. The conditional is true.

Converse: If the square $n^2$ of a natural number is odd, then $n$ is odd. The converse is true.

Since the conditional and its converse are true, the biconditional is true.
Determine if the biconditional is true. If false, give a counterexample.

An angle is a right angle iff its measure is 90°.

Conditional: If an angle is a right angle, then its measure is 90°. The conditional is true.

Converse: If the measure of an angle is 90°, then it is a right angle. The converse is true.

Since the conditional and its converse are true, the biconditional is true.
Check It Out! Example 3b

Determine if the biconditional is true. If false, give a counterexample.

\[ y = -5 \iff y^2 = 25 \]

Conditional: If \( y = -5 \), then \( y^2 = 25 \).

The conditional is true.

Converse: If \( y^2 = 25 \), then \( y = -5 \).

The converse is false.

The converse is false when \( y = 5 \). Thus, the biconditional is false.
In geometry, biconditional statements are used to write definitions.

A **definition** is a statement that describes a mathematical object and can be written as a true biconditional.
In the glossary, a **polygon** is defined as a closed plane figure formed by three or more line segments.
A **triangle** is defined as a three-sided polygon, and a **quadrilateral** is a four-sided polygon.
Think of definitions as being reversible. Postulates, however, are not necessarily true when reversed.
Example 4: Writing Definitions as Biconditional Statements

Write each definition as a biconditional.

A. A pentagon is a five-sided polygon.
   A figure is a pentagon if and only if it is a 5-sided polygon.

B. A right angle measures 90°.
   An angle is a right angle if and only if it measures 90°.
Write each definition as a biconditional.

4a. A quadrilateral is a four-sided polygon.
A figure is a quadrilateral if and only if it is a 4-sided polygon.

4b. The measure of a straight angle is 180°.
An $\angle$ is a straight $\angle$ if and only if its measure is 180°.
Biconditional Statements and Definitions

Lesson Quiz

1. For the conditional “If an angle is right, then its measure is 90°,” write the converse and a biconditional statement.
   Converse: If an \( \angle \) measures 90°, then the \( \angle \) is right.
   Biconditional: An \( \angle \) is right iff its measure is 90°.

2. Determine if the biconditional “Two angles are complementary if and only if they are both acute” is true. If false, give a counterexample.
   False; possible answer: 30° and 40°

3. Write the definition “An acute triangle is a triangle with three acute angles” as a biconditional.
   A triangle is acute iff it has 3 acute \( \angle \)s.
Warm Up
Solve each equation.

1. $3x + 5 = 17 \quad x = 4$
2. $r - 3.5 = 8.7 \quad r = 12.2$
3. $4t - 7 = 8t + 3 \quad t = -\frac{5}{2}$
4. $\frac{n + 8}{5} = -6 \quad n = -38$
5. $2(y - 5) - 20 = 0 \quad y = 15$
Objectives

Review properties of equality and use them to write algebraic proofs.

Identify properties of equality and congruence.
Vocabulary

proof
A **proof** is an argument that uses logic, definitions, properties, and previously proven statements to show that a conclusion is true.

An important part of writing a proof is giving justifications to show that every step is valid.
### Properties of Equality

<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition Property of Equality</td>
<td>If $a = b$, then $a + c = b + c$.</td>
</tr>
<tr>
<td>Subtraction Property of Equality</td>
<td>If $a = b$, then $a - c = b - c$.</td>
</tr>
<tr>
<td>Multiplication Property of Equality</td>
<td>If $a = b$, then $ac = bc$.</td>
</tr>
<tr>
<td>Division Property of Equality</td>
<td>If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.</td>
</tr>
<tr>
<td>Reflexive Property of Equality</td>
<td>$a = a$</td>
</tr>
<tr>
<td>Symmetric Property of Equality</td>
<td>If $a = b$, then $b = a$.</td>
</tr>
<tr>
<td>Transitive Property of Equality</td>
<td>If $a = b$ and $b = c$, then $a = c$.</td>
</tr>
<tr>
<td>Substitution Property of Equality</td>
<td>If $a = b$, then $b$ can be substituted for $a$ in any expression.</td>
</tr>
</tbody>
</table>
The Distributive Property states that
\[ a(b + c) = ab + ac. \]
Example 1: Solving an Equation in Algebra

Solve the equation $4m - 8 = -12$. Write a justification for each step.

\[
\begin{align*}
4m - 8 &= -12 & \text{Given equation} \\
\underline{+8} & \quad \underline{+8} & \text{Addition Property of Equality} \\
4m &= -4 & \text{Simplify.} \\
\frac{4m}{4} &= \frac{-4}{4} & \text{Division Property of Equality} \\
m &= -1 & \text{Simplify.}
\end{align*}
\]
Check It Out! Example 1

Solve the equation \( \frac{1}{2}t = -7 \) and write a justification for each step.

\[
\frac{1}{2}t = -7 \quad \text{Given equation}
\]

\[
2 \left( \frac{1}{2} \right) t = 2(-7) \quad \text{Multiplication Property of Equality.}
\]

\[
t = -14 \quad \text{Simplify.}
\]
Example 2: Problem-Solving Application

What is the temperature in degrees Fahrenheit \( F \) when it is 15°C? Solve the equation \( F = C \frac{9}{5} + 32 \) for \( F \) and justify each step.
Example 2 Continued

1. Understand the Problem

The answer will be the temperature in degrees Fahrenheit. List the important information:

\[ F = \frac{9}{5}C + 32 \]
\[ C = 15 \]
2-5 Algebraic Proof

Example 2 Continued

2 Make a Plan

Substitute the given information into the formula and solve.
Example 2 Continued

Solve

\[ F = \frac{9}{5}C + 32 \quad \text{Given equation} \]

\[ F = \frac{9}{5}(15) + 32 \quad \text{Substitution Property of Equality} \]

\[ F = 27 + 32 \quad \text{Simplify.} \]

\[ F = 59 \quad \text{Simplify.} \]

\[ F = 59^\circ \]
Look Back

Check your answer by substituting it back into the original formula.

\[ F = \frac{9}{5}C + 32 \]

\[ 59 = \frac{9}{5}(15) + 32 \]

\[ 59 = 59 \checkmark \]
Check It Out! Example 2

What is the temperature in degrees Celsius $C$ when it is $86^\circ F$? Solve the equation $C = \left(\frac{5}{9}(F - 32)\right)$ for $C$ and justify each step.
Understand the Problem

The answer will be the temperature in degrees Celsius. List the important information:

\[ C = \frac{5}{9}(F - 32) \quad F = 86 \]
Check It Out! Example 2 Continued

2. Make a Plan

Substitute the given information into the formula and solve.
Solve

\[ C = \frac{5}{9} (F - 32) \]  
Given equation

\[ C = \frac{5}{9} (86 - 32) \]  
Substitution Property of Equality

\[ C = \frac{5}{9} (54) \]  
Simplify.

\[ C = 30 \]  
Simplify.

\[ C = 30^\circ \]
Look Back

Check your answer by substituting it back into the original formula.

\[ C = \frac{5}{9}(F - 32) \]

\[
\begin{align*}
30 & = \frac{5}{9}(86 - 32) \\
30 & = 30 \checkmark
\end{align*}
\]
Like algebra, geometry also uses numbers, variables, and operations. For example, segment lengths and angle measures are numbers. So you can use these same properties of equality to write algebraic proofs in geometry.

Helpful Hint

$AB$ represents the length $AB$, so you can think of $AB$ as a variable representing a number.
Example 3: Solving an Equation in Geometry

Write a justification for each step.

\[ NO = NM + MO \]
\[ 4x - 4 = 2x + (3x - 9) \]
\[ 4x - 4 = 5x - 9 \]
\[ -4 = x - 9 \]
\[ 5 = x \]

- **Segment Addition Post.**
- **Substitution Property of Equality**
- **Simplify.**
- **Subtraction Property of Equality**
- **Addition Property of Equality**
Write a justification for each step.

\[
\begin{align*}
m\angle ABC &= m\angle ABD + m\angle DBC \\
8x^\circ &= (3x + 5)^\circ + (6x - 16)^\circ \\
8x &= 9x - 11 \\
-x &= -11 \\
x &= 11
\end{align*}
\]

\[\angle \text{Add. Post.} \quad m\angle ABC = 8x^\circ\]

\[\text{Subst. Prop. of Equality}\]

\[\text{Simplify.}\]

\[\text{Subtr. Prop. of Equality.}\]

\[\text{Mult. Prop. of Equality.}\]
You learned in Chapter 1 that segments with equal lengths are congruent and that angles with equal measures are congruent. So the Reflexive, Symmetric, and Transitive Properties of Equality have corresponding properties of congruence.
# 2-5 Algebraic Proof

## Properties of Congruence

<table>
<thead>
<tr>
<th>SYMBOLS</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reflexive Property of Congruence</strong></td>
<td></td>
</tr>
<tr>
<td>figure $A \cong$ figure $A$</td>
<td>$EF \cong EF$</td>
</tr>
<tr>
<td>(Reflex. Prop. of $\cong$)</td>
<td></td>
</tr>
<tr>
<td><strong>Symmetric Property of Congruence</strong></td>
<td></td>
</tr>
<tr>
<td>If figure $A \cong$ figure $B$, then figure $B \cong$ figure $A$.</td>
<td>If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.</td>
</tr>
<tr>
<td>(Sym. Prop. of $\cong$)</td>
<td></td>
</tr>
<tr>
<td><strong>Transitive Property of Congruence</strong></td>
<td></td>
</tr>
<tr>
<td>If figure $A \cong$ figure $B$ and figure $B \cong$ figure $C$, then figure $A \cong$ figure $C$.</td>
<td>If $PQ \cong RS$ and $RS \cong TU$, then $PQ \cong TU$.</td>
</tr>
<tr>
<td>(Trans. Prop. of $\cong$)</td>
<td></td>
</tr>
</tbody>
</table>
Remember!
Numbers are equal (\(=\)) and figures are congruent (\(\cong\)).
Example 4: Identifying Property of Equality and Congruence

Identify the property that justifies each statement.

A. \( \angle QRS \cong \angle QRS \) \hspace{1cm} \text{Reflex. Prop. of } \cong.

B. \( m\angle 1 = m\angle 2 \) so \( m\angle 2 = m\angle 1 \) \hspace{1cm} \text{Symm. Prop. of } =

C. \( \overline{AB} \cong \overline{CD} \) and \( \overline{CD} \cong \overline{EF} \), so \( \overline{AB} \cong \overline{EF} \). \hspace{1cm} \text{Trans. Prop. of } \cong

D. \( 32^\circ = 32^\circ \) \hspace{1cm} \text{Reflex. Prop. of } =
Identify the property that justifies each statement.

4a. $DE = GH$, so $GH = DE$.
4b. $94^\circ = 94^\circ$  
   Reflex. Prop. of $=$
4c. $0 = a$, and $a = x$. So $0 = x$.
   Trans. Prop. of $=$
4d. $\angle A \cong \angle Y$, so $\angle Y \cong \angle A$
   Sym. Prop. of $\cong$
Lesson Quiz: Part I

Solve each equation. Write a justification for each step.

1. \( \frac{z - 5}{6} = -2 \)

   \( \frac{z - 5}{6} = -2 \)  \hspace{1cm} \text{Given}

   \( z - 5 = -12 \)  \hspace{1cm} \text{Mult. Prop. of =}

   \( z = -7 \)  \hspace{1cm} \text{Add. Prop. of =}
Lesson Quiz: Part II

Solve each equation. Write a justification for each step.

2. \(6r - 3 = -2(r + 1)\)

\[
\begin{align*}
6r - 3 &= -2(r + 1) & \text{Given} \\
6r - 3 &= -2r - 2 & \text{Distrib. Prop.} \\
8r - 3 &= -2 & \text{Add. Prop. of } = \\
8r &= 1 & \text{Add. Prop. of } = \\
r &= \frac{1}{8} & \text{Div. Prop. of } =
\end{align*}
\]
Lesson Quiz: Part III

Identify the property that justifies each statement.

3. \(x = y\) and \(y = z\), so \(x = z\).  
   Trans. Prop. of =

4. \(\angle DEF \cong \angle DEF\)  
   Reflex. Prop. of \(\cong\)

5. \(AB \cong CD\), so \(CD \cong AB\).  
   Sym. Prop. of \(\cong\)
Warm Up
Determine whether each statement is true or false. If false, give a counterexample.

1. It two angles are complementary, then they are not congruent. 
   false; 45° and 45°

2. If two angles are congruent to the same angle, then they are congruent to each other. 
   true

3. Supplementary angles are congruent. 
   false; 60° and 120°
2-6 Geometric Proof

Objectives

Write two-column proofs.
Prove geometric theorems by using deductive reasoning.
Vocabulary

theorem
two-column proof
When writing a proof, it is important to justify each logical step with a reason. You can use symbols and abbreviations, but they must be clear enough so that anyone who reads your proof will understand them.
Example 1: Writing Justifications

Write a justification for each step, given that \( \angle A \) and \( \angle B \) are supplementary and \( m\angle A = 45^\circ \).

1. \( \angle A \) and \( \angle B \) are supplementary.  
   \[ m\angle A = 45^\circ \]  
   Given information

2. \( m\angle A + m\angle B = 180^\circ \)  
   Def. of supp \( \angle \)s

3. \( 45^\circ + m\angle B = 180^\circ \)  
   Subst. Prop of =  
   \( \textit{Steps 1, 2} \)

4. \( m\angle B = 135^\circ \)  
   Subtr. Prop of =
Helpful Hint

When a justification is based on more than the previous step, you can note this after the reason, as in Example 1 Step 3.
Check It Out! Example 1

Write a justification for each step, given that $B$ is the midpoint of $AC$ and $AB \cong EF$.

1. $B$ is the midpoint of $AC$.  
   Given information

2. $AB \cong BC$  
   Def. of mdpt.

3. $AB \cong EF$  
   Given information

4. $BC \cong EF$  
   Trans. Prop. of $\cong$
A **theorem** is any statement that you can prove. Once you have proven a theorem, you can use it as a reason in later proofs.
### Theorem

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2-6-1 Linear Pair Theorem</strong>&lt;br&gt;If two angles form a linear pair, then they are supplementary.</td>
<td>( \angle A ) and ( \angle B ) form a linear pair.</td>
<td>( \angle A ) and ( \angle B ) are supplementary.</td>
</tr>
</tbody>
</table>
**Theorem**

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2-6-2</strong> Congruent Supplements Theorem</td>
<td>$\angle 1$ and $\angle 2$ are supplementary. $\angle 2$ and $\angle 3$ are supplementary.</td>
<td>$\angle 1 \cong \angle 3$</td>
</tr>
</tbody>
</table>
A geometric proof begins with *Given* and *Prove* statements, which restate the hypothesis and conclusion of the conjecture. In a **two-column proof**, you list the steps of the proof in the left column. You write the matching reason for each step in the right column.
Fill in the blanks to complete the two-column proof.

**Given:** $XY$

**Prove:** $XY \cong XY$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $XY$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $XY = XY$</td>
<td>2. Reflex. Prop. of $=$</td>
</tr>
<tr>
<td>3. $XY \cong XY$</td>
<td>3. Def. of $\cong$ segs.</td>
</tr>
</tbody>
</table>
Check It Out! Example 2

Fill in the blanks to complete a two-column proof of one case of the Congruent Supplements Theorem.

**Given:** \( \angle 1 \) and \( \angle 2 \) are supplementary, and \( \angle 2 \) and \( \angle 3 \) are supplementary.

**Prove:** \( \angle 1 \cong \angle 3 \)

**Proof:**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. a. __<strong>?</strong></td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m\angle 1 + m\angle 2 = 180^\circ ) ( m\angle 2 + m\angle 3 = 180^\circ )</td>
<td>2. Def. of supp. ( \angle )</td>
</tr>
<tr>
<td>4. ( m\angle 2 = m\angle 2 )</td>
<td>4. Reflex. Prop. of =</td>
</tr>
<tr>
<td>5. ( m\angle 1 = m\angle 3 )</td>
<td>5. c. __<strong>?</strong></td>
</tr>
<tr>
<td>6. d. __<strong>?</strong></td>
<td>6. Def. of ( \cong \angle )</td>
</tr>
</tbody>
</table>

a. \( \angle 1 \) and \( \angle 2 \) are supp., and \( \angle 2 \) and \( \angle 3 \) are supp.

b. \( m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3 \)

c. Subtr. Prop. of =

d. \( \angle 1 \cong \angle 3 \)
Before you start writing a proof, you should plan out your logic. Sometimes you will be given a plan for a more challenging proof. This plan will detail the major steps of the proof for you.
### Theorems

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2-6-3 Right Angle Congruence Theorem</strong></td>
<td>$\angle A$ and $\angle B$ are right angles.</td>
<td>$\angle A \cong \angle B$</td>
</tr>
<tr>
<td>All right angles are congruent.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>2-6-4 Congruent Complements Theorem</strong></td>
<td>$\angle 1$ and $\angle 2$ are complementary. $\angle 2$ and $\angle 3$ are complementary.</td>
<td>$\angle 1 \cong \angle 3$</td>
</tr>
</tbody>
</table>
Helpful Hint

If a diagram for a proof is not provided, draw your own and mark the given information on it. But do not mark the information in the Prove statement on it.
Example 3: Writing a Two-Column Proof from a Plan

Use the given plan to write a two-column proof.

**Given:** \( \angle 1 \) and \( \angle 2 \) are supplementary, and \( \angle 1 \cong \angle 3 \)

**Prove:** \( \angle 3 \) and \( \angle 2 \) are supplementary.

**Plan:** Use the definitions of supplementary and congruent angles and substitution to show that \( m\angle 3 + m\angle 2 = 180^\circ \). By the definition of supplementary angles, \( \angle 3 \) and \( \angle 2 \) are supplementary.
### Example 3 Continued

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle 1$ and $\angle 2$ are supplementary. $\angle 1 \cong \angle 3$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $m\angle 1 + m\angle 2 = 180^\circ$</td>
<td>2. Def. of supp. $\angle s$</td>
</tr>
<tr>
<td>3. $m\angle 1 = m\angle 3$</td>
<td>3. Def. of $\cong \angle s$</td>
</tr>
<tr>
<td>4. $m\angle 3 + m\angle 2 = 180^\circ$</td>
<td>4. Subst.</td>
</tr>
<tr>
<td>5. $\angle 3$ and $\angle 2$ are supplementary</td>
<td>5. Def. of supp. $\angle s$</td>
</tr>
</tbody>
</table>
Use the given plan to write a two-column proof if one case of Congruent Complements Theorem.

**Given:** \( \angle 1 \) and \( \angle 2 \) are complementary, and 
\( \angle 2 \) and \( \angle 3 \) are complementary.

**Prove:** \( \angle 1 \cong \angle 3 \)

**Plan:** The measures of complementary angles add to 90° by definition. Use substitution to show that the sums of both pairs are equal. Use the Subtraction Property and the definition of congruent angles to conclude that \( \angle 1 \cong \angle 3 \).
### Check It Out! Example 3 Continued

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle 1$ and $\angle 2$ are complementary. $\angle 2$ and $\angle 3$ are complementary.</td>
<td>1. Given</td>
</tr>
</tbody>
</table>
| 2. $m\angle 1 + m\angle 2 = 90^\circ$  
$m\angle 2 + m\angle 3 = 90^\circ$ | 2. Def. of comp. $\angle s$ |
| 3. $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$ | 3. Subst.             |
| 4. $m\angle 2 = m\angle 2$                           | 4. Reflex. Prop. of $=$ |
| 5. $m\angle 1 = m\angle 3$                           | 5. Subtr. Prop. of $=$ |
| 6. $\angle 1 \cong \angle 3$ | 6. Def. of $\cong \angle s$ |
Write a justification for each step, given that $m\angle ABC = 90^\circ$ and $m\angle 1 = 4m\angle 2$.

1. $m\angle ABC = 90^\circ$ and $m\angle 1 = 4m\angle 2$  
   \[ \text{Given} \]

2. $m\angle 1 + m\angle 2 = m\angle ABC$  
   \[ \angle \text{ Add. Post.} \]

3. $4m\angle 2 + m\angle 2 = 90^\circ$  
   \[ \text{Subst.} \]

4. $5m\angle 2 = 90^\circ$  
   \[ \text{Simplify} \]

5. $m\angle 2 = 18^\circ$  
   \[ \text{Div. Prop. of =.} \]
2. Use the given plan to write a two-column proof.

**Given:** \( \angle 1, \angle 2, \angle 3, \angle 4 \)

**Prove:** \( m\angle 1 + m\angle 2 = m\angle 1 + m\angle 4 \)

**Plan:** Use the linear Pair Theorem to show that the angle pairs are supplementary. Then use the definition of supplementary and substitution.

1. \( \angle 1 \) and \( \angle 2 \) are supp.
   \( \angle 1 \) and \( \angle 4 \) are supp.

2. \( m\angle 1 + m\angle 2 = 180^\circ \), \( m\angle 1 + m\angle 4 = 180^\circ \)

3. \( m\angle 1 + m\angle 2 = m\angle 1 + m\angle 4 \)

1. Linear Pair Thm.

2. Def. of supp. \( \angle \)s

2-7 Flowchart and Paragraph Proofs

Warm Up

Lesson Presentation

Lesson Quiz
Warm Up
Complete each sentence.

1. If the measures of two angles are ____, then the angles are congruent. **equal**
2. If two angles form a ____?, then they are supplementary. **linear pair**
3. If two angles are complementary to the same angle, then the two angles are ____. **congruent**
Objectives

Write flowchart and paragraph proofs.
Prove geometric theorems by using deductive reasoning.
Vocabulary

flowchart proof
paragraph proof
A second style of proof is a **flowchart proof**, which uses boxes and arrows to show the structure of the proof.

The justification for each step is written below the box.
### Theorem 2-7-1: Common Segments Theorem

<table>
<thead>
<tr>
<th>THEOREM</th>
<th>HYPOTHESIS</th>
<th>CONCLUSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given collinear points $A, B, C, \text{ and } D$ arranged as shown, if $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$.</td>
<td>$\overline{AB} \cong \overline{CD}$</td>
<td>$\overline{AC} \cong \overline{BD}$</td>
</tr>
</tbody>
</table>

[Diagram of collinear points $A, B, C, \text{ and } D$]
Example 1: Reading a Flowchart Proof

Use the given flowchart proof to write a two-column proof.

Given: \( \angle 2 \) and \( \angle 3 \) are comp.
\( \angle 1 \equiv \angle 3 \)
Prove: \( \angle 2 \) and \( \angle 1 \) are comp.

Flowchart proof:
Example 1 Continued

Two-column proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 2 ) and ( \angle 3 ) are comp. ( \angle 1 \cong \angle 3 )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m\angle 2 + m\angle 3 = 90^\circ )</td>
<td>2. Def. of comp. ( \angle )s</td>
</tr>
<tr>
<td>3. ( m\angle 1 = m\angle 3 )</td>
<td>3. Def. of ( \cong \angle )s</td>
</tr>
<tr>
<td>4. ( m\angle 2 + m\angle 1 = 90^\circ )</td>
<td>4. Subst.</td>
</tr>
<tr>
<td>5. ( \angle 2 ) and ( \angle 1 ) are comp.</td>
<td>5. Def. of comp. ( \angle )s</td>
</tr>
</tbody>
</table>
Check It Out! Example 1

Use the given flowchart proof to write a two-column proof.

**Given:** \( RS = UV \), \( ST = TU \)

**Prove:** \( \overline{RT} \cong \overline{TV} \)

**Flowchart proof:**

1. \( RS = UV, ST = TU \)  
   - **Given**
2. \( RS + ST = TU + UV \)  
   - **Add. Prop of =**
3. \( RS + ST = RT, TU + UV = TV \)  
   - **Seg. Add. Post.**
4. \( RT = TV \)  
   - **Subst.**
5. \( \overline{RT} \cong \overline{TV} \)  
   - **Def. of \( \cong \) segs.**
### Check It Out! Example 1 Continued

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1.</strong> (RS = UV, ST = TU)</td>
<td><strong>1.</strong> Given</td>
</tr>
<tr>
<td><strong>2.</strong> (RS + ST = TU + UV)</td>
<td><strong>2.</strong> Add. Prop. of =</td>
</tr>
<tr>
<td><strong>3.</strong> (RS + ST = RT,)</td>
<td><strong>3.</strong> Seg. Add. Post.</td>
</tr>
<tr>
<td>(TU + UV = TV)</td>
<td></td>
</tr>
<tr>
<td><strong>4.</strong> (RT = TV)</td>
<td><strong>4.</strong> Subst.</td>
</tr>
<tr>
<td><strong>5.</strong> (\overline{RT} \cong \overline{TV})</td>
<td><strong>5.</strong> Def. of (\cong) segs.</td>
</tr>
</tbody>
</table>
Use the given two-column proof to write a flowchart proof.

**Example 2: Writing a Flowchart Proof**

**Given:** $B$ is the midpoint of $AC$.

**Prove:** $2AB = AC$

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $B$ is the midpoint of $AC$.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $AB \equiv BC$</td>
<td>2. Def. of mdpt</td>
</tr>
<tr>
<td>3. $AB = BC$</td>
<td>3. Def. of $\equiv$ segs.</td>
</tr>
<tr>
<td>5. $AB + AB = AC$</td>
<td>5. Subst.</td>
</tr>
<tr>
<td>6. $2AB = AC$</td>
<td>6. Simplify</td>
</tr>
</tbody>
</table>
Example 2 Continued

Flowchart proof:

- **Given**: \( B \) is the mdpt. of \( \overline{AC} \).
- **Def. of mdpt.**: \( \overline{AB} \cong \overline{BC} \)
- **Seg. Add. Post.**: \( AB + BC = AC \)
- **Def. of \( \cong \) segs.**: \( AB = BC \)
- **Subst.**: \( AB + AB = AC \)
- **Simplify.**: \( 2AB = AC \)
Check It Out! Example 2

Use the given two-column proof to write a flowchart proof.

Given: \( \angle 2 \cong \angle 4 \)
Prove: \( m\angle 1 \cong m\angle 3 \)

Two-column Proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 2 \cong \angle 4 )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 1 ) and ( \angle 2 ) are supplementary. ( \angle 3 ) and ( \angle 4 ) are supplementary.</td>
<td>2. Lin. Pair Thm.</td>
</tr>
<tr>
<td>3. ( \angle 1 \cong \angle 3 )</td>
<td>3. ( \cong ) Supps. Thm.</td>
</tr>
<tr>
<td>4. ( m\angle 1 = m\angle 3 )</td>
<td>4. Def. of ( \cong )</td>
</tr>
</tbody>
</table>
### Check It Out! Example 2 Continued

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle 2 \cong \angle 4$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle 1$ and $\angle 2$ are supplementary. $\angle 3$ and $\angle 4$ are supplementary.</td>
<td>2. Lin. Pair Thm.</td>
</tr>
<tr>
<td>3. $\angle 1 \cong \angle 3$</td>
<td>3. $\cong$ Supps. Thm.</td>
</tr>
<tr>
<td>4. $m\angle 1 = m\angle 3$</td>
<td>4. Def. of $\cong$</td>
</tr>
</tbody>
</table>

**Diagram:**

- $\angle 1$, $\angle 2$ are supp.
- $\angle 3$, $\angle 4$ are supp.
- Lin. Pair Thm.
- $\angle 2 \cong \angle 4$
- Given
- $\angle 1 \cong \angle 3$
- $\cong$ Supps. Thm.
- $m\angle 1 = m\angle 3$
- Def. of $\cong$  

*Holt Geometry*
A **paragraph proof** is a style of proof that presents the steps of the proof and their matching reasons as sentences in a paragraph. Although this style of proof is less formal than a two-column proof, you still must include every step.
# Theorems

<table>
<thead>
<tr>
<th><strong>THEOREM</strong></th>
<th><strong>HYPOTHESIS</strong></th>
<th><strong>CONCLUSION</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2-7-2</strong> Vertical Angles Theorem</td>
<td>$\angle A$ and $\angle B$ are vertical angles.</td>
<td>$\angle A \cong \angle B$</td>
</tr>
<tr>
<td>Vertical angles are congruent.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>2-7-3</strong> If two congruent angles are supplementary, then each angle is a right angle. ($\cong \triangle$ supp. $\rightarrow$ rt. $\triangle$)</td>
<td>$\angle 1 \cong \angle 2$ $\angle 1$ and $\angle 2$ are supplementary.</td>
<td>$\angle 1$ and $\angle 2$ are right angles.</td>
</tr>
</tbody>
</table>
Example 3: Reading a Paragraph Proof

Use the given paragraph proof to write a two-column proof.

**Given:** \( m\angle 1 + m\angle 2 = m\angle 4 \)

**Prove:** \( m\angle 3 + m\angle 1 + m\angle 2 = 180^\circ \)

**Paragraph Proof:** It is given that \( m\angle 1 + m\angle 2 = m\angle 4 \). \( \angle 3 \) and \( \angle 4 \) are supplementary by the Linear Pair Theorem. So \( m\angle 3 + m\angle 4 = 180^\circ \) by definition. By Substitution, \( m\angle 3 + m\angle 1 + m\angle 2 = 180^\circ \).
Two-column proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (m\angle 1 + m\angle 2 = m\angle 4)</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. (\angle 3) and (\angle 4) are supp.</td>
<td>2. Linear Pair Theorem</td>
</tr>
<tr>
<td>3. (m\angle 3 + m\angle 4 = 180^\circ)</td>
<td>3. Def. of supp. (\angle)s</td>
</tr>
<tr>
<td>4. (m\angle 3 + m\angle 1 + m\angle 2 = 180^\circ)</td>
<td>4. Substitution</td>
</tr>
</tbody>
</table>
Check It Out! Example 3

Use the given paragraph proof to write a two-column proof.

Given: $\angle WXY$ is a right angle. $\angle 1 \cong \angle 3$

Prove: $\angle 1$ and $\angle 2$ are complementary.

**Paragraph Proof:** Since $\angle WXY$ is a right angle, $m\angle WXY = 90^\circ$ by the definition of a right angle. By the Angle Addition Postulate, $m\angle WXY = m\angle 2 + m\angle 3$. By substitution, $m\angle 2 + m\angle 3 = 90^\circ$. Since $\angle 1 \cong \angle 3$, $m\angle 1 = m\angle 3$ by the definition of congruent angles. Using substitution, $m\angle 2 + m\angle 1 = 90^\circ$. Thus by the definition of complementary angles, $\angle 1$ and $\angle 2$ are complementary.
### Check It Out! Example 3 Continued

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle WXY ) is a right angle.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( m\angle WXY = 90^\circ )</td>
<td>2. Def. of right angle</td>
</tr>
<tr>
<td>3. ( m\angle 2 + m\angle 3 = m\angle WXY )</td>
<td>3. Angle Add. Postulate</td>
</tr>
<tr>
<td>4. ( m\angle 2 + m\angle 3 = 90^\circ )</td>
<td>4. Subst.</td>
</tr>
<tr>
<td>5. ( \angle 1 \cong \angle 3 )</td>
<td>5. Given</td>
</tr>
<tr>
<td>6. ( m\angle 1 = m\angle 3 )</td>
<td>6. Def. of ( \cong )( \angle ) s</td>
</tr>
<tr>
<td>7. ( m\angle 2 + m\angle 1 = 90^\circ )</td>
<td>7. Subst.</td>
</tr>
<tr>
<td>8. ( \angle 1 ) and ( \angle 2 ) are comp.</td>
<td>8. Def. of comp. angles</td>
</tr>
</tbody>
</table>
Example 4: Writing a Paragraph Proof

Use the given two-column proof to write a paragraph proof.

**Given:** ∠1 and ∠2 are complementary

**Prove:** ∠3 and ∠4 are complementary

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ∠1 and ∠2 are comp.</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. m∠1 + m∠2 = 90°</td>
<td>2. Def. of comp. ∠s</td>
</tr>
<tr>
<td>3. ∠1 ≡ ∠3, ∠2 ≡ ∠4</td>
<td>3. Vert. ∠s Thm</td>
</tr>
<tr>
<td>4. m∠1 = m∠3, m∠2 = m∠4</td>
<td>4. Def. of ≡ ∠s</td>
</tr>
<tr>
<td>5. m∠3 + m∠4 = 90°</td>
<td>5. Subst.</td>
</tr>
<tr>
<td>6. ∠3 and ∠4 are comp.</td>
<td>6. Def. of comp. ∠s</td>
</tr>
</tbody>
</table>
Paragraph proof:

\( \angle 1 \) and \( \angle 2 \) are complementary, so \( m\angle 1 + m\angle 2 = 90^\circ \) by the definition of complementary angles. \( \angle 1 \cong \angle 3 \) and \( \angle 2 \cong \angle 4 \) by the Vertical Angle Theorem. So \( m\angle 1 = m\angle 3 \) and \( m\angle 2 = m\angle 4 \) by the definition of congruent angles. By substitution, \( m\angle 3 + m\angle 4 = 90^\circ \), so \( \angle 3 \) and \( \angle 4 \) are complementary by the definition of complementary angles.
Check It Out! Example 4

Use the given two-column proof to write a paragraph proof.

Given: \( \angle 1 \cong \angle 4 \)
Prove: \( \angle 2 \cong \angle 3 \)

Two-column proof:

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \angle 1 \cong \angle 4 )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle 1 \cong \angle 2, \angle 3 \cong \angle 4 )</td>
<td>2. Vert. ( \triangle ) Thm.</td>
</tr>
<tr>
<td>3. ( \angle 2 \cong \angle 4 )</td>
<td>3. Trans. Prop. of ( \cong ) \text{Steps 1, 2}</td>
</tr>
<tr>
<td>4. ( \angle 2 \cong \angle 3 )</td>
<td>4. Trans. Prop. of ( \cong ) \text{Steps 2, 3}</td>
</tr>
</tbody>
</table>
Check It Out! Example 4 Continued

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\angle 1 \cong \angle 4$</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$</td>
<td>2. Vert. $\angle$ Thm.</td>
</tr>
<tr>
<td>3. $\angle 2 \cong \angle 4$</td>
<td>3. Trans. Prop. of $\cong$ $\text{Steps 1, 2}$</td>
</tr>
<tr>
<td>4. $\angle 2 \cong \angle 3$</td>
<td>4. Trans. Prop. of $\cong$ $\text{Steps 2, 3}$</td>
</tr>
</tbody>
</table>

**Paragraph proof:**

It is given that $\angle 1 \cong \angle 4$. By the Vertical Angles Theorem, $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. By the Transitive Property of Congruence, $\angle 2 \cong \angle 4$. Also by the Transitive Property of Congruence, $\angle 2 \cong \angle 3$. 
Lesson Quiz

Use the two-column proof at right to write the following.

1. a flowchart proof

   Given: \( \angle 1 \cong \angle 4 \)
   Prove: \( \angle 3 \cong \angle 2 \)

   \[ \begin{align*}
   \angle 1 \cong \angle 4 & \quad \text{Given} \\
   \angle 1 \cong \angle 2, \angle 3 \cong \angle 4 & \quad \text{Vert. \( \angle \) Thm.} \\
   \angle 4 \cong \angle 2 & \quad \text{Trans. Prop. of \( \cong \)} \\
   \angle 3 \cong \angle 2 & \quad \text{Trans. Prop. of \( \cong \)}
   \end{align*} \]

2. a paragraph proof

   It is given that \( \angle 1 \cong \angle 4 \). By the Vertical Angles Theorem, \( \angle 1 \cong \angle 2 \) and \( \angle 3 \cong \angle 4 \). By the Transitive Property of Congruence, \( \angle 4 \cong \angle 2 \) and \( \angle 3 \cong \angle 2 \).

Two-Column proof

1. \( \angle 1 \cong \angle 4 \) (Given)
2. \( \angle 1 \cong \angle 2, \angle 3 \cong \angle 4 \) (Vert. \( \angle \) Thm.)
3. \( \angle 4 \cong \angle 2 \) (Trans. Prop. of \( \cong \))
4. \( \angle 3 \cong \angle 2 \) (Trans. Prop. of \( \cong \))